

# Applications of Fuzzy Sets and Approximate Reasoning

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*Invited Paper*

*This paper discusses recent applications of fuzzy sets and the theory of approximate reasoning. The primary focus is on fuzzy logic control (FLC). We begin with a brief history of the key ideas, a survey of recent applications, and a discussion of the genesis of FLC in Japan. We then turn to a study of the general principles of FLC, considering it as a combination of ideas from conventional control theory, artificial intelligence, and fuzzy sets theory. We next provide a detailed analysis of a simple application in consumer electronics, namely, a fuzzy washing machine developed by Hitachi Corporation. In concluding sections we briefly consider other types of applications, including recent work on pilotless helicopters, fuzzy expert systems, and the concept of a fuzzy computer, and we discuss the potential for future developments. It is our opinion that the subject of FLC is still very much in its infancy, and that recent events mark the beginning of an entirely new genre of "intelligent" control.*

## I. INTRODUCTION

The subject of fuzzy sets was introduced by Lotfi A. Zadeh, University of California at Berkeley, in 1965 [1]. In that work Zadeh was implicitly advancing the thesis that one of the reasons humans are better at control than currently existing machines is that they are able to make effective decisions on the basis of imprecise linguistic information. Hence it should be possible to improve the performance of electromechanical controllers by modeling the way in which humans reason with this type of information.

The theory developed slowly at first, but by the early 1970's it had attracted a small international following. This included a number of westerners, mostly mathematicians, and a small number of Japanese engineers. The interest was

spurred primarily by intellectual curiosity, although even then there was a pervasive belief in the theory's ultimate applicability. During this time, investigations focused mainly on the mathematical properties of fuzzy sets and closely related notions, and numerous variants of fuzzy logic were explored. In 1971 [2] Zadeh introduced a theory of state equations for describing the behavior of fuzzy systems (systems whose descriptive parameters are fuzzy-valued), and in 1972 [3] he outlined the rationale for fuzzy control. An important landmark in this development was a 1973 paper [4], which introduced the basic notion of a linguistic variable, that is, a variable whose values are linguistic terms rather than numbers. For example, "Size" is a linguistic variable taking as values such terms as "small," "large," "not very large," etc. (see also [5], [6]). The concept of a linguistic variable in combination with the basic notion of a fuzzy IF-THEN rule, e.g., "IF pressure is very high, THEN volume is very low," played a major role in paving the way for applying the theory to real-world problems.

By the late 1970's, interest in fuzzy systems had grown rather explosively, attracting many researchers from around the world and spawning bibliographies with citations numbering in the thousands. Still, most of the work was theoretical. The main topics included fuzzy knowledge representation and reasoning schemes, the philosophical ramifications of fuzzy logic and fuzzy set theory, fuzzifications of various branches of classical mathematics, and several foundational challenges posed by probability theorists and the classical AI community. Partly in response to this, Zadeh put forth "possibility theory" [7] which showed how the fuzzy-sets model of natural language reasoning could be provided with an intuitively acceptable foundation.

Around this same time, there also emerged some important applications. One of these was fuzzy logic control (FLC). The genesis of FLC was a 1972 paper by Chang and Zadeh [8] outlining the basic approach, together with the 1973 paper mentioned above ([4]). The first implementation

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of these ideas was described in a seminal paper by Mamdani and Assilian of Queen Mary College, London, in 1975 [9] (see also [10], [11]), which demonstrated the viability of FLC for a small model steam engine. The first major commercial application of FLC was a temperature controller for a cement kiln, developed by Smidth and Co., Denmark [12]. Other early implementations of FLC were for a traffic junction controller [13] and a sludge controller for a wastewater treatment plant [14]. The first fuzzy inference chip was developed in the early 1980's at AT&T Bell Laboratories by Togai and Watanabe [15]. This was an implementation; using digital technology; of the mathematical Min and Max operations, which are used to represent intersection and union of fuzzy sets. The initial period (1971–1983) of the development of FLC is well overviewed in the annotated bibliography prepared by Tong [16].

The key principle underlying FLC is strikingly simple. In a conventional PID (Proportion, Integral, Derivative) controller the system being controlled is modeled analytically by a set of differential equations whose solution tells what adjustments should be made to the system's control parameters for each type of system behavior. The fuzzy logic controller, on the other hand, is based on a logical model which represents the thinking processes that a human operator might go through while controlling the system manually. At the Second IEEE Conference on Fuzzy Systems (San Francisco, CA, April 1993), Mamdani was awarded a fellowship in the IEEE in recognition for his being the first to implement the concept of FLC. In his acceptance speech, Mamdani stressed that this shift in focus—from modeling the system being controlled to modeling the thinking processes of the human controller—represents a fundamental paradigm shift in the theory of automatic control. Alongside neural nets and genetic algorithms, it stands as one of the first major steps into the era of "intelligent" control.

A highly readable and detailed history of the development of these ideas has recently appeared as [17]. Here we shall mention only two key events, which together served to trigger the very rapid and widespread propagation of interest in FLC. First was the opening of the first fully automated subway employing a fuzzy logic controller, in the city of Sendai in northern Japan. The new system controlled all aspects of accelerating to speed and braking for corners or stopping at platforms, so that the only human operator served essentially as a conductor, watching out for passengers' safety while getting on or off the train. Implementation of the Sendai controller is due to Yasunobu and Miyamoto of Hitachi's Systems Development Laboratory [18]–[20]. This work is a direct evolution of that of Mamdani and, like Mamdani's, it features an implementation of Zadeh's "compositional rule of inference" in software on a conventional digital computer. Through simulations, Yasunobu demonstrated that the fuzzy logic controller was superior to the conventional PID controller along several key parameters, including accuracy in stopping at platforms, rider comfort (jerkiness of acceleration and braking), and fuel economy. He proposed his ideas to Hitachi in 1979,

published his simulation results in 1985, and the Sendai Subway opened in 1987. The system was so successful that the city of Tokyo subsequently decided to use it for a new subway that is now nearing completion.

The second event was Takeshi Yamakawa's demonstration of his inverted pendulum experiment at the Second Congress of the International Fuzzy Systems Association (Tokyo, July 1987), see [21]–[25]. The inverted pendulum is a classic nonlinear control problem, wherein a pole is attached to a horizontal belt by a hinge, so that the pole can fall to the right or the left. The objective is to monitor the angular position and speed of the pole, and to move the belt to the right or left accordingly, so as to maintain the pole in an upright position. The problem becomes more difficult as the pole becomes shorter and/or is reduced in mass, since the required reaction times increase in proportion to the square of the amount by which either height or mass is decreased.

Yamakawa's controller featured two types of chips of his own design. One was a fuzzy rule chip, which directly implements the compositional rule of inference in hardware, and the other was a defuzzifier chip, which extracts a precise value from a fuzzy one (see the following sections for an elaboration of these ideas). An important feature of Yamakawa's approach is the use of analog techniques. This was done because the elementary operations employed in the compositional rule of inference, and for the most part also in the defuzzification operation, are the arithmetic Max and Min, which can be implemented so as to run much faster on an analog device. Another important feature was the introduction of parallelism. Each fuzzy inference rule was implemented on its own chip, so all chips could be fired in unison.

The controller Yamakawa presented in 1987 used 7 rule chips and 1 defuzzifier chip, and it demonstrated balancing response speeds approaching 100 times faster than those heretofore accomplished by a conventional PID controller. This controller only maintained vertical, and not horizontal, stability of the inverted pendulum (the pole remained vertical but would also creep to either the right or the left), however, whereas the classical problem entails both. But less than a year later, it was shown that by adding only 4 additional rule chips one could achieve both vertical and horizontal stabilization at the same speed as before. Since that time, Yamakawa has demonstrated the robustness of his system for nonlinear control by attaching a small platform to the top of the inverted pendulum, onto which a wine glass is placed, and the controller nicely compensated for the turbulence created as a liquid is poured into the glass. In another, even more dramatic demonstration, a small white mouse was placed on the platform, and the controller compensated for the totally erratic movements of the mouse.

#### *A. Recent Applications*

Many consumer products using fuzzy technology are currently available in Japan, and some are now being marketed in the US and Europe. It should be noted that

such applications of FLC do not typically use fuzzy inference chips, but instead use simulations through table look-ups on standard digital IC's. Such an approach is feasible where one does not require the speed of a specially dedicated fuzzy inference chip. As examples we may cite the following. Canon introduced a fuzzy logic controller in the autofocus mechanism of an 8 mm movie camera. The Matsushita (Panasonic) "Palmcorder" uses fuzzy logic for image stabilization. Each of Matsushita Electric, Hitachi, Sanyo, and Sharp now have their own "fuzzy washing machine," which automatically adjusts the washing cycle in response to various combinations of size of load, type of dirt (soil versus grease), amount of dirt, and type of fabric. In Matsushita's machine, the type and amount of dirt are detected by means of light sensors, which also use fuzzy controls. Other products using fuzzy control include vacuum cleaners, air conditioners, electric fans, kerosene heaters, microwave ovens, electric carpets, clothes driers, rice cookers, and hot plates. In addition to the companies listed above, the actors include Mitsubishi, Toshiba, Fujitsu General, Corona, and Toyotomi. A survey of recent applications to consumer products is presented in [26]. Two early industrial applications were for a large crane and for a road tunnel ventilation system, also designed by Yasunobu at Hitachi. A survey of recent industrial applications may be found in [27], and a constant flow of various types of applications is reported in the issues of the *Journal of Japan Society for Fuzzy Theory and Systems*, as well as in the proceedings of the Society's annual symposia.

It is useful to classify applications of FLC along the notion of "time lag" in the control cycle, insofar as this is associated with different types of control problems (see Section II for a discussion of time lag). For example, under Operational Control (short time lag) one may put

- air conditioning system
- anti-skid brakes
- arc welding robot
- auto-iris diaphragm for VTR
- automatic focusing for compact camera
- automatic transmission
- autonomous robot
- color adjustment of TV
- control for group of elevators
- dredging machine
- hot water supplier for shower
- speed control of automobile
- train control
- tunnel drilling

- and under Process Control (long time lag) one may put
- combustion control of refuse incineration plant
  - cement kiln
  - glass melting furnace
  - rain water pumping
  - tandem cold mill
  - tunnel ventilation
  - water purification process

These examples demonstrate the widespread utility of simple fuzzy logic controllers. It has been found that such

controllers have a definite advantage over the traditional PID controllers in that they work well even when the relation between the controller's input and output variables is nonlinear, they are much more robust with respect to degradation of any associated sensors, and they do not require redesign whenever one changes the controlled system's parameters, e.g., the desired engine speed. Classical PID controllers do not work well for the case of nonlinear control and, even for linear control, they typically must be designed anew whenever one resets the basic system parameters. In addition, and perhaps more importantly, fuzzy logic controllers are much easier to design than PID controllers, they require fewer electronic components, and are therefore cheaper to produce. It is for these reasons that many manufacturers now opt for FLC even in situations where PID controllers could be used. It may be said that the simplicity of FLC stems from its capacity to exploit one's tolerance for imprecision in control.

### B. The Genesis in Japan

We suggest that the very dramatic boom of interest in products using FLC came to Japan in 1990 and 1991 largely because Japanese consumer products at that time were ready, both in terms of technology and market trends, to make good use of fuzzy control theory, and because the Japanese were willing and able to utilize a corps of dedicated fuzzy set researchers. We believe that this is a better explanation than the somewhat widespread notion that Oriental minds are intrinsically fuzzy and Occidental minds are intrinsically not, although cultural differences might have played some role as well.

Whether due to such differences or simply differences in industrial practice, attention to detail in even small things is certainly cause for much of the Japanese market success in many types of consumer products. To Japanese industry no product is so mundane or insignificant that it does not deserve the best engineering with as much creative use as possible of the latest technologies. This has been the custom in Japan for at least the past twenty-five years, and it is perhaps even more true of products made specifically for the Japanese market than it is for exported products.

This was clearly true of the use of microprocessor technology. When microprocessors became widely available in the mid to late 1970's and knowledge of how to use them in various applications became widespread there was clearly a difference in perceptions between American and Japanese engineers in how they might be used in the near future in applications other than low-cost computers. American engineering was developing relatively expensive commercial applications such as high-precision surveying equipment or military applications. Certainly, there was talk also of consumer product applications, but it centered around very major products such as luxury automobiles as soon as noise shielding technology could be improved enough to allow microprocessors in cars. Japanese engineers, on the other hand, were already very excited about putting microprocessors even in small appliances.

During the 1980's it became common in the domestic Japanese home appliance market to make products as small as electric rice cookers, sometimes retailing for less than US\$100, microprocessor-based. Certainly American consumers are familiar with all of the consumer electronic export-oriented products Japanese industry was making. These are of course available in Japan too, but it might surprise some Americans to see just how many seemingly mundane home appliances there are in Japan that use the same technology. Obviously, this was a major stimulus for Japanese chip making technology, but there were other results as well. Powerful information processing ability is of little use without good input, so there was good reason to develop better low-cost sensors of all types to use with the low-cost processing chips, particularly for home appliances.

Thus, by the late 1980's, Japanese industry was basing even many small appliances on both microprocessors and new sensors, though there was perhaps not always a clear idea of how to use so much processing power. The situation could be described as a powerful hardware technology just waiting for new software concepts to drive it. Fuzzy control theory was an absolutely perfect fit.

One of the virtues of FLC is that it can be implemented quite easily on general-purpose digital electronic hardware. Clearly, there are potential advantages in special-purpose fuzzy inference hardware, particularly for a more direct representation of membership functions and large numbers of inferences based thereon. Thus there is good justification for the work of Togai, Yamakawa, and others in this direction, and it is likely that special-purpose hardware will play a more central role in future fuzzy set applications, even small-scale ones, in the future. But fuzzy control does not face the same limitations as some other approaches such as neural networks. Neural networks can be implemented or "simulated" in software on general-purpose computing devices, but in many applications such implementations are simply too inefficient to be useful. Hence, further development of some types of neural network applications seem to be quite dependent on further developments in massively parallel hardware. Comparatively, FLC has been much less dependent on special hardware. Just as conventional control theory is compatible with implementations in analog electronics, FLC is compatible with general-purpose, von Neuman architecture, digital computing electronics.

Japanese engineers began contributing to the theory of fuzzy sets and systems as early as 1968. During the late 1970's and throughout the 1980's researchers at leading Japanese engineering schools implemented various demonstration systems in laboratories, and later developed actual commercial applications such as waste treatment plants and the Sendai subway train control system mentioned earlier. Some academics encouraged even their undergraduate engineering students to develop demonstration-type fuzzy control applications as their thesis projects for their bachelor degrees. Furthermore, as the academic interest and a few large applications lent credibility to the promise of fuzzy set applications, various research organizations were established for further development that involved large-scale

cooperation between government, industry, and academia; an institutional structure common in Japan. The results of all these factors were increasing numbers of engineers knowledgeable in fuzzy engineering and increasing interest on the part of industry.

In hindsight it seems almost obvious that the widespread use of fuzzy control in Japanese consumer products was certain to occur. With microprocessors and sensors already used in so many appliances and with the increasing knowledge of and interest in fuzzy control it was a very easy match. As is typical in microprocessor-based device development, one merely develops the program based on the logic chip to be used, tests the results by programming it onto a PROM, and then goes into mass production etching the program onto mask ROM chips. For a good programmer familiar with microprocessor technology it is really little harder than implementing the same algorithms in software on a large computer-based system.

The Japanese mass market also makes the fuzzy control product boom seem inevitable in hindsight. Some Japanese describe their countrymen as *nessiyasuku sameyasui*, easily heated up and easily cooled down, a sort of boom and bust mentality for new ideas. Japan is a highly urbanized, proudly uniform, highly informed, and very status-conscious society. Japanese consumers, even during recessions, are willing to be less price-conscious than Americans, and in return they demand not only high quality, but also more exciting new technological twists than the average American consumer does. The forever ubiquitous advertising slogan in Japan is simply the word *shinseihin*, new product. As soon as one manufacturer began advertising "fuzzy" as the latest new idea in technology it became absolutely essential for all competitors to follow suit if they were to have any hope of retaining market share. This is basically what happened around 1990, reaching full pitch in 1991. In that year the appliance departments of the typical department store had stickers on more than half of the products on the floor saying simply "fuzzy," everything from tiny rice cookers to large washing machines and refrigerators. From a mass marketing standpoint, of course, the term "fuzzy" was appropriated merely to take advantage of the latest fad, and already in 1993, it is difficult to find a new product labeled simply "fuzzy." One now sees labels for "neuro fuzzy" or "neuro & fuzzy," signifying an even more sophisticated type of product, so that the idea of labeling a product as "fuzzy" alone seems very much passé.

The actual application of FLC to consumer and other products is nonetheless clearly just beginning, and research is ongoing toward further extending this approach as well as combining it with classical control. Some of these issues are discussed in the sections that follow.

## II. THE RATIONALE AND PRINCIPLES OF FUZZY LOGIC CONTROL

In this paper it is assumed that the reader is familiar with the basic theory of fuzzy sets, such as presented in the original paper by Zadeh [1]. We refer here to the classical

or Mamdani approach to FLC, by which we mean strictly speaking a more recent slight modification of the original approach as developed by Mamdani and Assilian [9], and which is the basis for most practical approaches still in use today. We begin by discussing the classical approach as existing within the intersection of the following three conceptual frameworks:

- 1) Conventional control theory, particularly its general concepts and structures.
- 2) Artificial intelligence, particularly (heuristic) rule-based knowledge representation and inference techniques.
- 3) Fuzzy set theory, particularly its use in representing linguistic variables and fuzzy inference.

The early and continued technical success of FLC has been the result of the ability of researchers to combine these three fields in such a way as to take advantage of their respective strengths while avoiding their weaknesses.

#### *A. Conventional Control—Its Strengths and Weaknesses*

Most automatic devices designed to control even moderately complex systems are based on feedback. Feedback is simply the concept that an output of the process or system to be controlled is measured by some type of sensing device, and if the measured value differs from the desired value for that output then the control device will actuate some corrective action on one of the system's inputs in such a way as to attempt to correct the "error" in the output.

The concept of feedback control is so intuitively straightforward that many people assume it must be quite simple to implement in the form of an automatic control device, but the reality is that to design a truly effective automatic controller is often the greatest challenge an engineer must face. A major cause of the difficulty is time lag. There is always some lag in the controlled process before its outputs respond as desired to changes in its inputs, and there may also be significant lags in the actions of the control device itself, or in its sensors or actuators. For these reasons, feedback control means in practice that the system is constantly trying to correct for conditions not as they are now, but as they existed sometime earlier.

When we design an automatic control device of the feedback type there will be several characteristics we will desire it to have, but the problems raised by time lag make it very difficult to obtain all of these characteristics simultaneously. If there is some sudden change in the system due to extraneous noise then we will probably desire our control device to correct the system output levels as quickly as possible by making a forceful adjustment in the desired direction. But if we try too hard to design a control device with that type of responsiveness then we are likely to find that the device has made the system behavior unstable. That is, the device may attempt to correct errors so forcefully that it results in causing the system to oscillate wildly above and below the desired level. Furthermore, even if we somehow manage to design the control device with satisfactory levels of responsiveness and stability, we

may still find that it lacks accuracy, meaning that in steady-state conditions the system output may have a tendency to remain for long periods somewhat above or below the desired level. Due to problems of this sort, it is nearly impossible to design an effective automatic control device based only on a simple minded application of the negative feedback principle.

Fortunately, as we look a little deeper into control theory, we find some hope for dealing effectively with these problems. In the case of conventional control theory the solution comes primarily from two directions. First, conventional control theory has a very highly developed methodology for designing and adjusting automatic control devices based on mathematical models of the systems they are to control [28], [29]. Secondly, on closer examination we find that there are actually a variety of structural options to choose from in determining the level of corrective action the device should make given the error conditions. Obviously, the level of corrective action should be in some way proportional to the error, but specifically what about the error should this be? In the simplest type of feedback control it is directly proportional simply to the instantaneous error level itself. This is called proportional, or P, control. One alternative to proportional control is to make the corrective action proportional in part to the instantaneous error, but add to that a term proportional to the derivative of the error. This is called proportional plus derivative, or PD, control. Because the derivative indicates the direction of change in error it has a short-term anticipatory effect. The result is that PD control often makes it possible to increase system responsiveness and stability simultaneously.

Another option is to add a term for integrating the error over some time interval. Because this adds emphasis to small but persisting errors its major benefit is that it can allow for increases in steady-state accuracy without significantly increasing instability under any conditions. If the integrative term is added to proportional control it becomes PI control, or if added to PD control, the result is called PID control for proportional plus integrative plus derivative.

Much of the development of conventional control coincides with the age of analog electronics, and the typical implementation of a conventional PID control device is in analog circuitry. The integral and derivative elements are approximated in analog fashion, each element is given an appropriate gain, and the result is summed electronically.

Although PD, PI, and PID control are often useful alternatives to proportional control, it is generally still necessary to have a good methodology for designing the control device if there is to be any hope of it being effective in its task. In the case of conventional control theory this methodology centers around developing a mathematical model of the process to be controlled and then applying various analytical techniques to that model to determine the characteristics of an automatic device which might effectively control the process. These analytic techniques are highly evolved, having developed over many decades of research and application.

To begin with the most basic similarities between fuzzy and conventional control, nearly all examples of fuzzy control are also based primarily on the feedback structure. Furthermore, the manner of specifying the desirable characteristics for a control device and the manner of measuring performance according to those characteristics are typically the same in the fuzzy control world as they are in conventional control.

In a slightly less obvious way, fuzzy control theory also borrowed from the conventional theory the options of adding derivative and integral elements to the proportional feedback. Even among the earliest prototypes developed by Mamdani *et al.*, there are examples of fuzzy control that are analogous to the PD type of feedback control because the control actions are based on their instantaneous values as well as changes in the variables.

Few would dispute the premise that conventional control theory, being based on precise mathematical models of the processes to be controlled, is a sophisticated science. It is very effective in designing control devices for certain types of applications, but it nonetheless has certain limitations.

The first limitation is that conventional control theory seems to work best when one is attempting to design a control device for a process that can be well approximated by a model with linear and otherwise straightforward relationships between just a few variables. This is not to say that conventional control theorists have no interest in nonlinear systems, but at best conventional nonlinear control is considered an advanced topic well beyond the comprehension of the average undergraduate engineering student, and at worst it is unworkable in practice. Therefore, if we find that fuzzy control works well in controlling processes with complex, nonlinear relationships between several variables then clearly that is one advantage over conventional control.

A second limitation, related to the first, is that conventionally designed automatic control devices often have relatively narrow performance bands. This is because many types of real-world processes and systems can be well approximated by linear models only if the variables are limited to rather narrow ranges. Under ordinary conditions, the process can be assumed to stay within these ranges, but if due to extraordinary conditions a variable does go beyond this range then the control device may no longer be able to make appropriate adjustments.

Clearly, the practical implications of these two limitations are significant. Until recently, certain types of processes were still controlled by human operators simply because it proved too difficult to devise an automatic control device by the conventional methods. This was due not to the lack of sensors or to any other hardware problem, but simply because of nonlinearities or other complexities in the processes. Some examples are trash burning plants and waste treatment plants.

In addition, conventional control theory is difficult to learn. Many engineering students find control theory to be very challenging even when limited only to relatively simple linear systems. Although this level of difficulty

might be a cause for pride among experts in conventional control theory, it is reasonable to consider it an overall disadvantage. Experience shows that it is relatively easy to reach a basic level of competence in the design of fuzzy control devices, and we consider that to be another important advantage fuzzy control can offer over the conventional theory. Moreover, fuzzy control systems tend to require considerably less developing time (less manpower) than their conventional counterparts and are thus more cost-effective.

### *B. Expert Systems Technology and its Applicability to Control*

For the past decade, expert systems or knowledge-based systems have become quite popular, and there are good reasons for that popularity. It is debatably the first approach to artificial intelligence that is both powerful and truly practical. In addition, it is quite easy to learn enough about expert systems technology to be able to apply it well. Because expert systems are now quite well known, it will be useful to our purposes here to explain the similarities they have with what we are calling the classical approach to fuzzy control.

The first premise of expert systems technology is that, for some types of complex systems, human intuition and practical experience are often better guides than the best "hard" mathematical analysis available for understanding the behavior of the system in question. The second premise is that it is often possible to express most of that intuitive, experience-based, heuristic knowledge in a set, relatively simple format. Several such formats are possible, but the simplest and most used is just a collection of a few dozen to a few hundred IF-THEN rules. The third premise is that once the knowledge is expressed in a knowledge base of some type of uniform format, it is then possible to program a computer to automatically draw conclusions from that knowledge and from the facts as they are immediately available. In fact, the process of inferring the conclusions can usually be left to a general-purpose inference engine, which means that to develop a new expert system it is often necessary only to develop the knowledge base and to use this along with commercially available expert systems software.

Despite this simplicity, expert systems technology has proven very powerful in certain types of applications. Some expert systems have captured in a few hundred rules nearly all of the expert knowledge of highly paid professionals in a given field. Some examples include physicians who are experts in diagnosing certain categories of diseases or geologists who are expert at predicting where it is best to drill for oil. Because it is based on intuition and experiential human knowledge, there is no reason that it should be any harder to apply the technology to nonlinear systems than it is to linear ones. In the technological sense, expert systems technology is very simple especially if commercially available expert system development software is used. The greatest challenge of expert systems is of a psychological or epistemological

nature: it is often difficult to extract knowledge from a human expert's mind and express it in a simple format. This has come to be known as the "knowledge acquisition bottleneck."

All of the characteristics we have just described for expert systems are also true for the Mamdani approach to FLC. Fuzzy control also attempts to capture intuition and experience in the form of IF-THEN rules, and to automatically draw conclusions from them and from the facts at hand. There are only two major differences between FLC and the ordinary type of expert system. First, they are different in the nature of their applications. Expert systems are ordinarily used for diagnosis, design, or some other role that could be described as consultation, which means they continuously interface with humans. FLC of course is typically used for automatic control, which means that there should be little need for human interface during normal operation. The second difference is more subtle. Many expert systems do make use of fuzzy logic or some other schema for expressing uncertainty, but many others do not. In some applications expert systems seem to work very well without any expression of vagueness or uncertainty. On the other hand, heuristic rule-based control simply does not seem to work well without the concept of the linguistic variable that comes from fuzzy set theory. In fact, at times it has been the custom to use "linguistic control" as the preferred synonym for fuzzy control. The use of linguistic (fuzzy) variables implies that we must also use some form of fuzzy inference.

A steam engine controller, for example, may attempt to make adjustments to both the boiler heat source and the throttle based on measurements of sensors for boiler pressure and for engine speed. If we wish to base this controller on a PD structure then we should consider two variables each for pressure and speed. We will consider both pressure error (the amount above or below the desired pressure) and pressure change (the direction and degree of change in pressure since the last measurement); similarly, for speed error and speed change. The fuzzy controller of Mamdani and Assilian for this application contains 24 rules of the following type (14 for the heater and 9 for the throttle):

IF the Pressure Error is a Large Negative value,  
 or a Moderate Negative value,  
 or a Small Negative value;  
 and  
 the Pressure Change is not a Large Positive value;  
 and  
 the Speed Error is a Large Negative value;  
 and  
 the Speed Change is not a Large Positive value  
 THEN the Heat Adjustment should be Moderate in the Positive direction.

When one actually designs a fuzzy controller it may be more convenient to use a more concise format for stating the rules. To demonstrate a typical form of rule notation we will restate the same sample rule. The abbreviations for the variables could be as follows:

Table 1 Heater Adjustment Rules for a Steam Engine Controller

		Pressure Error						
		NB	NM	NS	ZO	PS	PM	PB
P r e s s u r e	NB	PB	PB	PB	PM	PM	PS	ZO
	NM	PB	PB	PM	PM	PS	ZO	NS
	NS	PB	PM	PM	PS	ZO	NS	NM
	ZO	PM	PM	PS	ZO	NS	NM	NM
C h a n g e	PS	PM	PS	ZO	NS	NM	NM	NB
	PM	PS	ZO	NS	NM	NM	NB	NB
	PB	ZO	NS	NM	NM	NB	NB	NB

SE = Speed Error  
 PE = Pressure Error  
 SC = Speed Change  
 PC = Pressure Change  
 HC = Heat Change (adjustment to the heat level)  
 TC = Throttle Change (adjustment to the throttle).

Similar abbreviations can be used for the values

PB = Positive Big (a large positive value)  
 PM = Positive Medium  
 PS = Positive Small  
 ZO = near Zero  
 NS = Negative Small  
 NM = Negative Medium  
 NB = Negative Big.

The rule in question can then be written as

IF (PE = NB OR PE = NM OR PE = NS) AND  
 NOT(PC = PB) AND  
 SE = NB AND  
 NOT(SC = PB)  
 THEN HC = PM.

It has also become customary to summarize all of the rules in tables, and this is particularly convenient if there are only two sensed variables. For example, if we were to simplify the previous illustration so that heater adjustments were to depend only on pressure error and pressure change then we could easily express our rules in one small table such as Table 1. This is a particularly precise format, it readily ensures that there are rules applicable for every conceivable circumstance, and it is a convenient format for computer input.

A more important issue than notation is just what one means by such statements as "the Pressure Error is a Large Negative value." This is the subject of the next section. What we wish to point out at this time is that it is possible that a person experienced in the operation of a steam engine could express his knowledge of how to control the engine with a list of heuristics of this type. Furthermore, this list of heuristics would look very much like the knowledge base of a rule-based expert system. Finally, since the mind of a human operator seems to function on the basis of

such heuristics it is conceivable that this type of heuristic knowledge could suffice for an automatic control device as well, and in that case there would be no need for precise mathematical models.

### C. Linguistic Variables in Fuzzy Control

Let us first consider one further difference between an automatic control device based on heuristic knowledge and some types of expert systems. Many expert systems concern themselves only with discrete phenomena. It is often said that there is no such thing as being a little pregnant, and so it is with many of premises and conclusions dealt with in some expert systems. It may be argued that even with discrete phenomena it may be more realistic to allow for uncertainty both in the facts and in the rules themselves, and this is why some expert systems make use of confidence factors or some other method of stating uncertainty. It may also be argued that some phenomena should be taken as fuzzy numbers. One example might be the symptom HIGH FEVER in a medical diagnostic expert system. But it is by no means obvious that vagueness is essential to all expert systems.

However, an automatic control device based on intuitive knowledge is quite another matter. For a rule in a control device, normally, both the premise and the conclusion will deal with continuous variables. (We consider ON-OFF control a special case which we will not discuss here.) Ordinarily, in a practical problem concerning the relationship between two or more variables, one thinks first of using functions. But a moment of reflection will reveal that mathematical functions in the ordinary  $y = f(x)$  form are usually not a natural way to express intuitive or experiential knowledge. For example, even an engineer when driving an automobile rarely thinks of stating a precise function for how many degrees the steering wheel must be turned at a certain speed in order to correct the course of the automobile so many degrees to the right or the left. At the very least, mathematical functions as we normally see them are a very unnatural way to express intuitive knowledge. Clearly, rules of the type already described are a much more natural way to express this knowledge, but we must carefully define our premises and conclusions. Precisely what do we mean by "the pressure error is a large, negative value"?

It may seem reasonable to simply divide the continuous range of possible pressure error values into precise, fixed intervals. We could say, for example, that  $-5$  PSI or lower is a large negative value, a value of  $-3$  to  $-5$  PSI is a moderate negative value, and so on. But there are two reasons that fixed intervals are not the best form of expression here. The first is the general psychological rationale for linguistic (fuzzy) variables in the first place. The human mind seems to naturally define and process information in a vague way, and this is reflected in the ways we tend to use language. It seems unnatural to think of "a tall man" as any man precisely 182 cm or taller, for that would mean that a man 182.0 cm is tall whereas

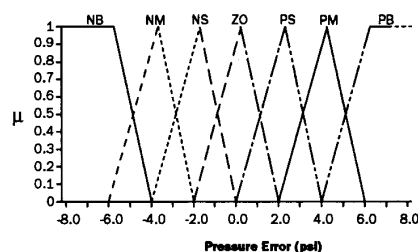


Fig. 1. Sample membership functions for a steam engine controller.

one 181.9 cm is not tall. It would be just as unnatural to think in terms of precise intervals about values of pressure error when attempting to describe rules about it. The second reason that precisely fixed intervals are undesirable in rule-based control is performance related. Clearly, a controller based on such a scheme would tend to behave erratically in some circumstances because a small change in control device inputs could result in large changes in its control actions.

The best way available for dealing with these problems is fuzzy set theory, and so it is clear at last why the concept of control devices based on intuitive knowledge rather than models leads us naturally to something that is called fuzzy control. The values NB (Negative Big), NM, NS, ZO, PS, PM, and PB for PE (Pressure Error) could be defined as the fuzzy sets (more specifically, fuzzy numbers) whose membership functions are as shown in Fig. 1. Because it takes as values linguistic terms which are further represented as fuzzy numbers, the variable PE can alternatively be described as a linguistic variable or as a "fuzzy variable." The possible values for PC, SE, SC, HC, TC, and whatever other input and output variables occur in a given control application can all be defined in similar ways, and thus all of the variables are linguistic variables.

The defining membership functions for all variables would most naturally be based on our intuitive assumptions, just as the rules themselves are. We must know what we will mean when we say "IF the pressure error is a large negative value ...." Thus in practice, the definitions of the values of the various variables usually precedes the statement of the rules. The membership functions can and sometimes do take some nonlinear form (as in [1]), but it seems now to be more common to use piecewise-linear functions, either triangular or trapezoidal, as in Fig. 1. Fuzzy control devices based on triangular and trapezoidal functions seem to perform as well in practice as those based on some other form of membership functions, and in many cases the triangular functions are easier to implement. Recently, it has become fairly common to assume triangular fuzzy numbers in researching other potential applications of fuzzy sets and fuzzy arithmetic. It would be an interesting psychological problem to investigate whether or not triangular membership functions are as accurate as some

other forms in expressing our intuitive concepts of fuzzy numbers.

Having said that rule-based control should be based on linguistic variables, we must now consider the mechanisms for fuzzy inference. Many variations are possible, and one can make this problem as simple or as complex as one likes. Those interested in the theoretical aspects of this question can describe whole classes of possible definitions for the fuzzy equivalents of various logical operators and their mathematical basis. For example, the classes of fuzzy conjunctions and disjunctions are closely related to the mathematical theory of  $T$ -norms and  $T$ -conorms [30]. But it is also true that fuzzy control seems in practice to work remarkably well even when based on the most straightforward concepts possible for fuzzy inference as were used by Mamdani, and as were suggested starting from the original Zadeh 1965 paper [1].

Questions related to finding the best forms of fuzzy inference to use in fuzzy control and various other applications has been seriously debated since the early 1970's. Currently, the most common opinion seems to be that, although the Mamdani methods do work well, other forms of conjunction and disjunction may result in improved performance. For the sake of simplicity, let us first assume the Mamdani classical approach in the following explanation of FLC.

The complete process of inference from rules based on linguistic variables can be broken into four parts:

- 1) The "fuzzification" of input values.
- 2) The evaluation of a compound premise containing various primitive premises joined by conjunctions, disjunctions, and negations.
- 3) The formation of overall fuzzy outputs based on all of the rules.
- 4) The "defuzzification" of those fuzzy outputs to form crisp final output values.

The first two steps are easily illustrated by the sample rule stated above for a steam engine controller. The raw inputs of the control device (values obtained from the measured values of one or more of the outputs of the process to be controlled) will typically be crisp values, simply real numbers stated to some degree of precision. Let us refer to these crisp values as follows:

$v_{PE}$  = pressure error = current measured pressure – predetermined desirable pressure

$v_{PC}$  = pressure change = current measured pressure – previous measured pressure

$v_{SE}$  = speed error = current measured speed – predetermined desirable speed

$v_{SC}$  = speed change = current measured speed – previous measured speed

The process of fuzzifying the inputs is very straightforward because it comes directly from the membership functions as defined. Because we are using the most common definitions, Max, Min, and subtraction from 1, for the fuzzy equivalents of the logical AND, OR, and NOT, respectively, we can

evaluate the premise

$$(PE = NB \text{ OR } PE = NM \text{ OR } PE = NS) \text{ AND} \\ \text{NOT}(PC = PB) \text{ AND } SE = NB \text{ AND} \\ \text{NOT}(SC = PB)$$

as

$$p = \text{Min} \{ \text{Max} [\mu_{PE=NB}(v_{PE}), \mu_{PE=NM}(v_{PE}), \\ \mu_{PE=NS}(v_{PE})], [1 - \mu_{PC=PB}(v_{PC})], \\ \mu_{SE=NB}(v_{SE}), [1 - \mu_{SC=PB}(v_{SC})] \}$$

where, e.g.,  $PE = NB$  refers to the fuzzy set NB defined on the scale for PE. The graph of the fuzzy set  $A$ , which represents the conclusion (in this case, the set PM for HC), is truncated at this scalar value  $p$ , to obtain fuzzy set  $B$ . (This is illustrated by a specific example in Section IV.)

However, the rule base typically contains several rules relating to the same output variable. For example, a fuzzy steam engine controller may contain several rules with HC in their conclusions. Therefore, to obtain the overall output in its fuzzy state, the usual procedure is to evaluate the premise for each rule, truncate the fuzzy set in the conclusion of that rule at that scalar value, and then take the union of the resulting fuzzy sets for all of the relevant rules. Thus if we index the rules 1 to  $n$ , if we let  $p_i$  represent the evaluation of the premise of rule  $i$  in the way we described above, and if Output Variable =  $A_i$  is the conclusion of rule  $i$ , and  $B_i$  is the fuzzy set obtained by truncating the graph of  $A_i$  at  $p_i$ , then the final value or Output Value,  $B^*$ , is the fuzzy set defined as

$$B^* = \bigcup_{i=1}^n B_i.$$

The original and most natural way to define fuzzy set union operator is by taking the maximum of the component set membership functions for each element. Therefore, for all  $y$  in the real range of the output variable

$$\mu_{B^*}(y) = \text{Max}_{i=1}^n [\mu_{B_i}(y)].$$

All that remains is to defuzzify this output value. A control action cannot be performed if it is left in the form of a fuzzy set so somehow it must be converted to an appropriate crisp value. Some of the earliest fuzzy controllers used the "mean of maxima" method, meaning that the crisp output was taken as the value at which the membership function of the fuzzy set attained its maximum, but if there were multiple maxima the mean of these was taken. This was easy to implement, but it was not so intuitively appealing as the self-explanatory centroid or "center of gravity" method (see Section IV), used in most later fuzzy controllers. In fact, there is a range of defuzzification methods that is bounded by the mean of maxima and the mean of support methods (see [31]).

### III. THE FLC DEVELOPMENT PROCESS

We can briefly review the principles of fuzzy control by considering the development process for an automatic control device based on the classical approach. We can break the process into various logical steps. The first step is to define the process interface. This is very little different from the first step one would make in the development of a conventional control device. It requires the designer to determine which process attributes should be sensed and used as inputs to the control device as well as the particular sensors to be used. Similarly, the designer determines which process variables can be adjusted by the control device and chooses appropriate actuators.

Step two is to define the exact nature of the variables in their crisp state. For a variable such as pressure error, what will be the desired value to which we will compare the measured value? Will we use a related variable, pressure change? What other related variables might be used? These points may be quite straightforward, but they are nonetheless important to the success of the control device.

A third step is to define the fuzzy variable descriptions for each variable input to the control device and for each variable by which the control device will attempt to control the process. It is important that the various membership functions should be well considered because otherwise it would be quite difficult to develop useful rules in the next step. Therefore, it is necessary at this stage to have a good intuitive feel for the nature of the variables.

The fourth step is probably the most crucial one because this involves developing the rules themselves. If there is some history of controlling the process in question with human operators then probably most of the rules will be based on the knowledge of one or more expert operators. If this is not the case, then the engineer designing the control device might experiment with the process enough so that he or she has a reasonably good intuitive feel for the process, and then will attempt to develop the rules alone. In either case, and as always in knowledge engineering, this is a difficult task.

As step five, the designer must decide on the precise nature of the inference methodology to be used, including the method of defuzzification for the control variables. This might be as described in the previous section, but many alternatives are available as we already suggested. At this stage, the designer must also begin to consider the detailed algorithms for implementing those inference rules in order to proceed smoothly into the next phase.

The sixth step is the implementation itself. With modern digital hardware, this is quite simple, as we illustrate in the next section.

As the seventh and final step we consider the testing and fine-tuning of the control device. Here the task is to select the membership functions for the fuzzy sets that will serve as meanings for the linguistic terms appearing in the inference rules. This is a relatively weak aspect of FLC, at least in the case of the classical approach, because the design is not based on analysis in the usual sense. Normally,

for any type of system designed totally on the basis of intuitive knowledge, fine-tuning amounts to observing the results and then intuitively adjusting membership functions that were derived from intuition in the first place. For simple controllers, this trial-and-error method normally serves adequately. As the number of rules grows large, however, this approach to selecting an acceptable collection of membership functions becomes less feasible.

The problem of how to fine-tune more complex controllers has nonetheless met with a practical solution through a unique application of neural nets. Here one employs a form of back propagation, wherein a neural net learns the needed membership functions from a set of training examples. The idea was developed originally by Takagi and Hayashi at Matsushita Electric Company's Central Research Laboratory [32]–[35], and then later replicated by Maeda and others at Hitachi's System Development Laboratory [36]. As a test case, Maeda and his coworkers applied this technique to the development of a controller that had been designed previously by trial and error. Using this technique, they were able to accomplish in one month what had formerly taken six. There has also been advances in using genetic algorithms for this task [37]–[45].

Fuzzy controllers offer the designer a flexibility in making choices at three levels, each of which may be a subject of subsequent tuning:

- 1) a choice of representing linguistic terms by fuzzy sets,
- 2) a choice of representing logic operations from well-defined classes of possible operations (e.g.,  $T$ -norms and  $T$ -conorms for logical conjunction and disjunction),
- 3) a choice of the defuzzification method from a well-defined range of possible methods.

This flexibility is an important asset of fuzzy controllers. It gives the designer the opportunity to tailor his approach as may be required by the context of each application.

### IV. EXAMPLE OF FLC IN A HOME APPLIANCE

Two well-known early consumer electronics applications of FLC are the image stabilizer in Matsushita Corporation's hand-held video camera [46] and the new genre of "fuzzy" or "fuzzy-neuro" washing machines. This section illustrates the basic techniques of FLC in the simple example of a fully automatic washing machine developed by Hitachi Corporation [47].

The objectives of Hitachi's development team were to save water, to decrease noise, to preserve cloth quality, to increase the effectiveness of washing, to shorten the washing time, and to do this in the simplest manner possible. Their approach was to simulate the "veteran" housewife, and to this end they tried to capture in a set of inference rules the veteran's experience in doing washing on manually controlled machines. Inputs to the controller are amount of clothes, quality of clothes, amount of dirt, and type of dirt, i.e., soil versus grease, and outputs of the controller are strength of churning of the rotor blades

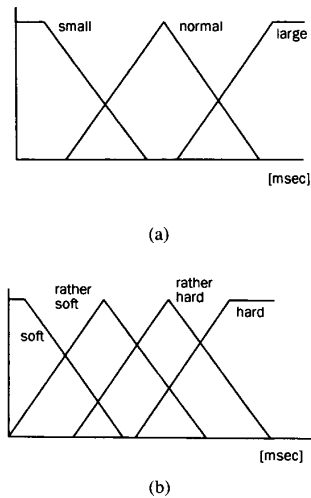


Fig. 2. Inputs to the controller. (a) Amount of clothes. (b) Quality of clothes.

and length of the washing period. The discussion in [47] describes the basic control method in terms of the first two input parameters only, and it describes only a one-time setting of the output variables for each washing load. From personal communication with the paper's primary author, however, it was learned that the actual machine does use measurements of the amount and type of dirt in a type of adaptive control, wherein the output parameters are continuously being readjusted during the washing cycle. Moreover, it was learned that the adaptive part of the control process uses a type of neural net. The details of these aspects were regarded as proprietary, but we were told that in order to measure amount and kind of dirt, Hitachi's machine uses electrical conductivity techniques. This may be contrasted with Matsushita's fuzzy washer, which uses a specially designed photo-optical sensor, and with Sanyo's washer, which uses a thermal sensor. A general discussion of the use of neural nets in combination with FLC may be found in [48].

#### A. Amount and Quality of Clothes

Determination of the amount and the quality of the clothes to be washed is accomplished by a "two-stage inverse dynamo method." In the first stage, a "little" water is let into the washing tub, and the motor is powered on, thereby turning the rotor in the tub and causing the water and clothes to start rotating. Then the motor is turned off, and the inertia of the clothes and water creates a force on the blades of the rotor, causing the motor to continue rotating. This turns the motor into a dynamo, generating a small amount of electrical power. The length of time during which power is generated in this way is measured, and the measurement is used as an indicator of the amount of clothes: a larger amount produces greater inertia, leading to longer generation time. Thus the amount of clothes can be directly represented as a collection of fuzzy sets whose universe of discourse is this generation time. "Small amount,"

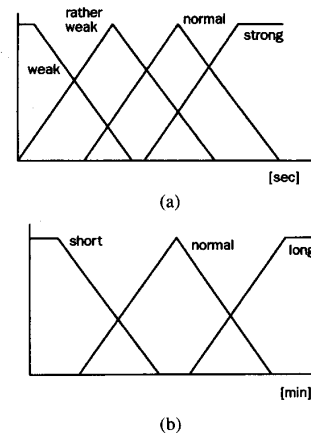


Fig. 3. Outputs from the controller. (a) Strength of churning. (b) Length of washing.

"normal amount," and "large amount" are represented with Z-shaped, triangular, and S-shaped membership functions, respectively, as shown in Fig. 2(a).

The appropriate amount of washing water can then be inferred from the amount of clothes. In the second stage, the same procedure is applied as above, but now with a "suitable" amount of water. This time, when the rotor is turned, the clothes can spread out in the water more freely, and "soft" clothes will thus create less inertia than "hard" clothes. Again the power generation time is measured, and then this measurement is compared with the one obtained above. Because there is now more water, the second measurement should be larger than the former. A small difference between the two indicates "soft" clothes, and a large difference indicates "hard" clothes. Thus the quality of clothes can be represented as a collection of fuzzy sets whose universe of discourse is this difference. "Soft," "rather soft," "rather hard," and "hard" are represented by membership functions as shown in Fig. 2(b).

#### B. Selection of Washing Method

An acceptable, if not actually optimal, washing method can be inferred from the quality and the amount of clothes. In [47], these relationships are expressed in a set of 12 rules having the forms:

*Rule 1:* If the amount of clothes is small and the quality is soft, then water churning is weak and washing time is short.

*Rule 12:* If the amount of clothes is large and the quality is hard, then water churning is strong and washing time is long.

with the complete set described as in Table 2.

Membership functions for the outputs from the controller are shown in Fig. 3. In the washer, the churning motion is achieved by powering the motor for, say,  $t$  seconds in the clockwise direction, then breaking power for  $t$  seconds, then powering in the counterclockwise direction for  $t$  seconds, then breaking power again for  $t$  seconds, and so on. Hence the strength of churning is determined by the amount of

**Table 2** Rules for Fuzzy Washing Machine

		Amount of Clothes		
		Small	Normal	Large
Quality of Clothes	Soft	weak short	rather weak short	normal normal
	Rather soft	rather weak short	normal normal	normal normal
	Rather hard	rather weak short	normal normal	normal long
	Hard	rather weak short	normal normal	strong long

time the power is left on in each direction. This means that churning strength may be given as in Fig. 3(a) by fuzzy sets for “weak,” “rather weak,” “normal,” and “strong,” defined on a time domain measured in seconds. Figure 3(b) shows fuzzy sets for washing time, given as “short,” “normal,” and “long,” defined on a time domain measured in minutes.

These 12 rules serve to illustrate the basic approach. From our personal communication with that paper’s primary author, however, it further was learned that, for size of load, there was actually an extra level, “rather large,” between “normal” and “large,” and for quality of clothes there was an extra level, “medium,” between “rather soft” and “rather hard.” Thus the actual controller was based on a 5-row, 4-column table, signifying 20 rules. Moreover, since each rule has two conclusions, one for churning strength and one for washing time, this yielded initially a total of 40 rules. By eliminating redundancies (i.e., “don’t cares” in certain premises), these 40 rules were then reduced to a final set of 19. Details of these last steps were also regarded as proprietary. Here it suffices to know that the final set of 19 rules divides into two groups, one dealing with churning strength and one dealing with washing time.

The operation of the controller, as explained in terms of the above 12 rules, is depicted in Fig. 4. Here one has as an exemplary  $i$ th rule, “If the amount of clothes is large and the quality of clothes is rather hard, then water churning is strong and washing time is long,” and an exemplary  $j$ th rule, “If the amount of clothes is normal and the quality of clothes is rather hard then water churning is normal and washing time is normal.” Inputs to the controller  $x_1$  and  $x_2$  are the above-mentioned measurements of power generation time and difference in power generation times. For the given rule  $i$ , one determines

$$\text{Min} [\mu_{\text{large\_amount}}(x_1), \mu_{\text{rather\_hard\_quality}}(x_2)]$$

and derives as conclusions the fuzzy sets obtained by truncating  $\mu_{\text{strong\_churning}}$  and  $\mu_{\text{long\_time}}$  at this minimum. Similarly, for the given rule  $j$ , one truncates  $\mu_{\text{normal\_churning}}$  and  $\mu_{\text{normal\_time}}$  at

$$\text{Min} [\mu_{\text{normal\_amount}}(x_1), \mu_{\text{rather\_hard\_quality}}(x_2)].$$

This same process is applied as well to the other 10 rules. Then the unions  $B_1^*$  and  $B_2^*$  of all the truncated (derived)

fuzzy sets corresponding, respectively, to churning strength and washing time are formed, as shown at the bottom of Fig. 4. The final outputs from the controller  $y_1$  seconds for each direction of churning, and  $y_2$  minutes of washing, are computed by finding the centroids, respectively, of the regions bounded by  $\mu_{B_1^*}$  and  $\mu_{B_2^*}$ . The general formula is

$$y_k = \frac{\int B_k^*(y)y dy}{\int B_k^*(y) dy}$$

where  $y$  ranges over the appropriate time domain, and  $k = 1$  or  $2$ .

### C. Implementation

It is stated in [47] that the final 19 rules were implemented on a multi-tasking 4-b microprocessor as a set of 19 lookup tables, but the exact nature of these tables is not discussed. Nonetheless it does not seem difficult to imagine how such tables might be built. First, divide the two input domains and the two output domains into discrete units, consisting, say, of 10 points each. Then for each rule (now having only one concluding term), construct a 100-row, 12-column table as follows. The first two columns contain all possible pairs of input points (each pair thereby indexing a row), and, for each input pair (row), the remaining 10 columns contain the grades of membership  $\mu_B(y)$  of the fuzzy set  $B$  representing the corresponding derived (truncated) conclusion, i.e.,  $y$  ranges over the associated conclusion domain. Such tables can also be reduced considerably by eliminating redundancies, these being “don’t cares” in one or the other of the two components in input pairs.

Then, for each pair of inputs to the controller, all tables are accessed in unison. Assume for the sake of the discussion that there are 9 rules for churning strength and 10 rules for washing time. Then to obtain a membership function for the above-mentioned union  $B_1^*$ , say, one looks at the 9 rows selected from the 9 relevant tables, and, for each of columns 3 through 12, runs down that column through all 9 rows, taking the maximums. This immediately yields a point-by-point definition of  $B_1$ . Then the centroid can be computed using the discrete version of the formula given above

$$y_k = \frac{\sum_{i=1}^{10} B_k^*(y_i)y_i}{\sum_{i=1}^{10} B_k^*(y_i)}$$

where the  $y_i$  are the discrete points in the relevant time domain.

While there are many variants on this approach, the one described here is typical of those used in most consumer electronics applications. Because of their evident simplicity and “designability,” one can easily incorporate such controllers without significantly increasing the overall cost of the product.

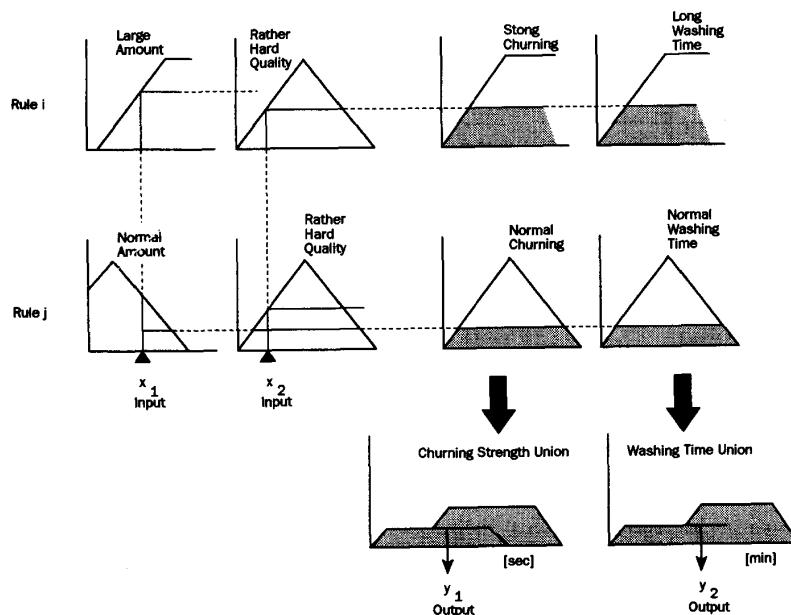


Fig. 4. The fuzzy inference method.

## V. PROSPECTS FOR THE FUTURE OF FLC

Very recently, much more complex fuzzy logic controllers have appeared. An ambitious project is the voice-controlled helicopter developed by M. Sugeno at Tokyo Institute of Technology. Here the objective is to develop a helicopter which responds to voice commands like "hover," "forward," "right," "up," "land," etc., where each such operation is fully automated via FLC. Sugeno successfully accomplished all functions with a 1-m model and then moved to a 3-m model [49], [50]. Video tapes demonstrating the mastery of all these functions on the larger model were presented by Sugeno at the Second IEEE Conference on Fuzzy Systems (San Francisco, CA, April 1993). Hovering is well-known to be a difficult stability problem; beginning helicopter pilots typically train for weeks before being able to do this manually. Hence successfully automating this operation is in itself a very impressive result.

A recent opinion among Japanese researchers is that most of the important theoretical work in FLC has now been completed and that the next step is up to the manufacturers, that is, to develop applications. This is reflected, for instance, in the fact that FLC was one of the three major areas of focus in the original program at the Laboratory for International Fuzzy Engineering Research (LIFE, established by a consortium of Japanese industries and governmental agencies for the period 1989–1995), whereas the subject is barely mentioned within its current program. What is perhaps more correctly the case, however, is that the initial stage of theoretical development is now more or less complete, and that moving to the next stage will require solving an assortment of substantially more difficult problems.

The possibilities for future work, leading to far more sophisticated logic-based controllers, are nonetheless clear. This will amount to moving from simple one-step rule-based systems to systems employing multistep reasoning—i.e., rule chaining, together with automatic truth maintenance systems—and which are integrated with other knowledge representation, reasoning, and learning paradigms. Thus advancing to this next type of "intelligent" control will require progress in a number of key subareas.

Although most applications to date have been produced in Japan, there has been a rapidly expanding growth of interest in the United States. Togai Infralogic, based in California, is now a major producer of fuzzy logic chips, boards, and other hardware in collaboration with Motorola. As mentioned earlier, M. Togai and H. Watanabe are credited with designing the first fuzzy inference chip [15]. Another US company, Apronix, San Jose, CA, made its debut in 1992 with a fuzzy logic controller that stabilizes a double pendulum (an inverted pendulum consisting of two poles attached together with a hinge) and demonstrated in simulation that it is theoretically possible to stabilize even a triple pendulum [51]. This was intended as a show piece demonstration; it is generally held that balancing a double pendulum is not theoretically possible with classical PID control. These and several other US companies are now developing applications of FLC for American-made products.

In addition to fuzzy inference chips, there are now some hardware implementations of the concept of a "fuzzy neuron," with applications to pattern recognition [52], [53]. Also of interest are recent experiments that implement fuzzy inference in an optical device [54].

## VI. OTHER APPLICATIONS OF FUZZY SETS

Although fuzzy sets have thus far been employed in consumer electronics almost exclusively in terms of fuzzy controllers, as discussed in the previous sections, their potential for consumer electronics is far greater. Our aim in this section is to overview other applications of fuzzy sets, distinct from fuzzy controllers, which are likely to find their way into computer electronics in the future.

One domain in which fuzzy sets and the associated approximate reasoning have already established great utility is the domain of expert systems. Although fuzzy controllers belong to this domain as well, as explained earlier in this paper, they represent only one special area within it. Fuzzy expert systems have been investigated since the mid 1970's, but actual implementations began emerging only in the 1980's. Examples of some early fuzzy expert systems are: SPERIL-II, designed for damage assessment of existing civil engineering structures [55]; CADIAG-2, designed for medical diagnosis in internal medicine [56]; RUBRIC, designed for full text document retrieval [57]; and FAULT, designed for financial analysis [58]. Although research on approximate reasoning is still very active [59], many specific fuzzy expert systems or expert system shells are now available, too many to be reviewed here (see [60]–[62] for further information). In Japan, for example, Hitachi Corporation markets a fuzzy expert system shell, ES/KERNEL, which by mid 1991 sold over 2000 copies.

Closely connected with fuzzy expert systems are fuzzy databases. The motivation for the use of fuzzy set theory in the design of databases lies in the need to handle information that is incomplete, contradictory, imprecise, or otherwise deficient, and to allow users to formulate queries in natural language. Research in this area began in the early 1980's (see, e.g., [63]–[65]) and continues to be very active [66]. Fuzzy set theory has also been applied in information retrieval [67], which is closely connected with databases.

One of the earliest and most promising applications of fuzzy set theory is in the areas of pattern recognition and clustering. These applications seem both natural and appropriate due to the fact that many of the categories we commonly encounter and employ have imprecise boundaries. Various subareas are subsumed under this heading, including speech recognition, handwritten character recognition, image processing, scene analysis, and medical diagnosis. The whole area of fuzzy pattern recognition and clustering, which has been under development since the early 1980's, is well covered by three monographs [68]–[70] and an edited volume that contains virtually all seminal papers in this area [71].

Fuzzy set theory has already made an important contribution to the broad area of decision making. Fuzziness allows the decision maker to frame the goals and constraints in vague, linguistic terms, which are more realistic in reflecting the usual state of knowledge and preferences involved. Fuzzy methods are especially suitable for multiobjective, multiperson, or multistage decision making. Fuzzy decision making was already well developed in the late 1970's [72].

The current literature in this area is prolific, as exemplified by five representative books [62], [73]–[76].

It is interesting that the idea of fuzzy switching circuits and automata was well developed theoretically already in the late 1970's [77], but it took more than a decade before fuzzy hardware became commercially available. Hardware chips implementing fuzzy inference rules, defuzzification, and other operations needed for implementing fuzzy systems are now offered by Omron, Motorola, and other vendors. One of the main projects of LIFE in Japan is to develop a computer based on hardware that would allow for both fuzzy and classical (binary) information processing [78], [79].

Most of the mentioned applications of fuzzy sets can also find utility in robotics, as demonstrated in several research laboratories in Japan. For example, Hirota at Hosei University has done extensive work with fuzzy control in robots. One is a robot which plays a two-dimensional ping-pong game [80]; another is an arm which grasps at irregularly moving object [81]; and one of the more dramatic results is a robot which throws darts at an object falling through an array of pegs, like in a pinball machine, and scoring a hit on virtually every try. The latter uses a general-purpose, programmable device, which it marketed commercially by Mitsubishi. Other work in this area is that of Ishikawa at IBM Japan, who has produced an autonomous robot which uses fuzzy control for obstacle avoidance. The robot avoids both stationary and moving obstacles, and it uses fuzzy control in particular for sensing the sizes and shapes of objects. Further applications are robots for making Japanese flower arrangements, for Chinese character calligraphy, and for inspection of plant seedlings.

Fuzzy set theory has been applied in numerous other areas, e.g., economics [82], psychology [83], behavior and social science [84], industrial engineering [85], risk analysis [86], earthquake research [87], systems identification and modeling [88], [89], engineering design [90], and regression analysis [91], but it is not the purpose of this paper to give a comprehensive coverage of all documented applications of fuzzy set theory. A recent tutorial on FLC is [92]. At the end of the references section we have included a few additional books and a list of journals that cover extensively various applications of fuzzy set theory.

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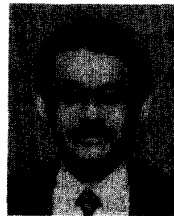
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